MODELING AND SIMULATION OF A THREE PHASE INDUCTION MOTOR UNDER OPEN PHASE FAULT

H. Zaimen[1*], A. Rezig[2], S. Touati[3***], I. Bouaissi[4]**

[1] Department of Electrical Engineering, University of Jijel, Algeria
[2] Department of Electrical Engineering, University of Jijel, Algeria
[3] Nuclear Research Centre of Birine, Djelfa; Algeria
* hichamlamel@gmail.com
** ali.rezig.@gmail.com
***saidtouati@yahoo.fr

Abstract. *This paper discusses a contribution to the modeling of a three-phase induction motor with one of its stator phases, open. The dynamic behavior of motor for different operation modes (healthy and faulty) will be studied by using the equivalent two-phase d-q model referenced to the stationary reference frame.*
All results of this work were obtained using the MATLAB software.

Keywords: *Three-phase induction motor; Open phase fault; Two-phase dq-model.*

1. Introduction

The induction motor is present in several industrial applications. The massive use of these IMs is mainly due to their low manufacturing cost, their reasonably high efficiency and their robustness requiring little maintenance. They are reliable in operations but are subject to different types of undesirable faults.

One of the most known faults in stator windings of induction motors is open circuit. With one phase open-circuited, an induction motor can continue to be operated with an asymmetrical winding and unbalanced excitation. However, the loss of one phase will drastically change the dynamic behaviour of the motor because the interactions between the lost phase and the rest of the motor phases due to mutual coupling no longer exist [1].

The most widely used technique for modelling induction motors is the equivalent two-phase model (d-q model). Due to the asymmetrical structure of a three-phase IM with a disconnected phase, the two-phase d-q model of a healthy three-phase IM is different from faulty three-phase IM model [2].

To show the open phase fault effect on the motor performances, the two-phase model is simulated using Matlab/ M-File software.

2. Mathematical models of IM

A. Induction Motor Model in the Park Reference Frame

The three-phase model for both stator and rotor of an induction motor is described in matrix form by equations (1)-(4):

- *Voltage equations:*

$$\left[v_{s\_abc}\right]=\left[R_s\right]\left[i_{s\_abc}\right]+\frac{d}{dt}\left[\Phi_{s\_abc}\right] \quad (1)$$

$$\left[v_{r\_abc}\right]=\left[R_r\right]\left[i_{r\_abc}\right]+\frac{d}{dt}\left[\Phi_{r\_abc}\right] \quad (2)$$

- *Flux-currents equations:*

$$\left[\Phi_{s\_abc}\right]=\left[L_{ss}\right]\left[i_{s\_abc}\right]+\left[L_{sr}\right]\left[i_{r\_abc}\right] \quad (3)$$

$$\left[\Phi_{r\_abc}\right]=\left[L_{rs}\right]\left[i_{s\_abc}\right]+\left[L_{rr}\right]\left[i_{r\_abc}\right] \quad (4)$$

$[v_{s\_abc}]$, $[i_{s\_abc}]$ and $[\Phi_{s\_abc}]$, are respectively voltage, current and flux vectors for stator.

For the rotor windings: $[v_{r\_abc}]$, $[i_{r\_abc}]$ and $[\Phi_{r\_abc}]$.

The stator resistance matrix $[R_s]$ and rotor resistance matrix $[R_r]$ define as follows:

$$\left[R_s\right]_{3\times3}=r_s\left[I\right]_{3\times3}; \quad \left[R_r\right]_{3\times3}=r_r\left[I\right]_{3\times3}$$

In equations (3)-(4), $[L_{ss}]$, $[L_{rr}]$ are stator and rotor winding inductance matrix, and $[L_{sr}]$, $[L_{rs}]$ are stator to rotor mutual inductance and rotor to stator mutual inductance matrix respectively. All these inductances are defined as [2]:

$$\left[L_{ss}\right]_{3\times3}=l_{ls}\left[I\right]_{3\times3}+l_{ms}\begin{bmatrix}1 & -0.5 & -0.5\\ -0.5 & 1 & -0.5\\ -0.5 & -0.5 & 1\end{bmatrix} \quad (5)$$

$$\left[L_{rr}\right]_{3\times3}=l_{lr}\left[I\right]_{3\times3}+l_{ms}\begin{bmatrix}1 & -0.5 & -0.5\\ -0.5 & 1 & -0.5\\ -0.5 & -0.5 & 1\end{bmatrix} \quad (6)$$

$$[L_{sr}]_{3\times3} = l_{ms}\begin{bmatrix} cos(\theta) & cos\left(\theta+\frac{2\pi}{3}\right) & cos\left(\theta-\frac{2\pi}{3}\right) \\ cos\left(\theta-\frac{2\pi}{3}\right) & cos(\theta) & cos\left(\theta+\frac{2\pi}{3}\right) \\ cos\left(\theta+\frac{2\pi}{3}\right) & cos\left(\theta-\frac{2\pi}{3}\right) & cos(\theta) \end{bmatrix} \quad (7)$$

$$[L_{rs}]_{3\times3} = [L_{sr}]^T$$

In equations (5)-(7) [2]:

l_{ls}, l_{ms} are stator leakage and magnetizing inductance;

l_{lr} is the rotor leakage inductance;

θ: rotor angular position (see Fig.1).

The two-phase model of IM is carried out by Park's transformation (as shown in Fig.1). Where:

d: is the direct axis, **q**: quadratic axis.

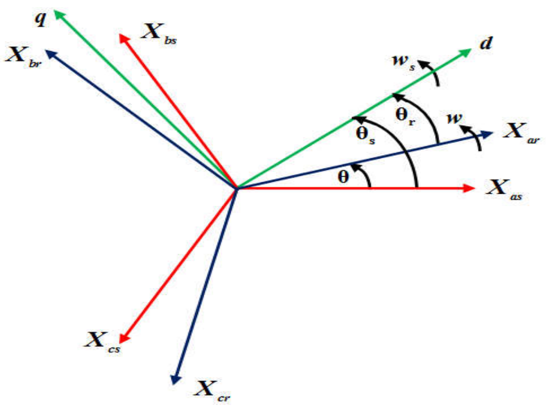


Fig.1. Axis transformation (from 3-phase to 2-phase).

For the development of the motor model, a Park reference frame is assumed to be lined up with the magnetic field and to rotate at the same speed ω_s [3].

The Park transformation is directly expressed by the following matrix product [3]:

$$[X_{dqo}] = [P]\,[X_{abc}] \quad (8)$$

Where:

$$[P] = \sqrt{\frac{2}{3}}\begin{bmatrix} cos(\psi) & cos\left(\psi-\frac{2\pi}{3}\right) & cos\left(\psi+\frac{2\pi}{3}\right) \\ -sin(\psi) & -sin\left(\psi-\frac{2\pi}{3}\right) & -sin\left(\psi+\frac{2\pi}{3}\right) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix} \quad (9)$$

Applying the Park transformations to flux, currents and voltages in equations (1)-(4), we obtain:

- *For stator windings:*

$$[v_{s\_dq0}] = [P_s][R_s][P_s]^{-1}[i_{s\_dq0}] + [P_s]\frac{d}{dt}\left\{[P_s]^{-1}[\Phi_{s\_dq0}]\right\} \quad (10)$$

$$[\Phi_{s\_dq0}] = [P_s][L_{ss}][P_s]^{-1}[i_{s\_dq0}] + [P_s][L_{sr}][P_s]^{-1}[i_{r\_dq0}] \quad (11)$$

- *For rotor windings:*

$$[v_{r\_dq0}] = [P_r][R_r][P_r]^{-1}[i_{r\_dq0}] + [P_r]\frac{d}{dt}\left\{[P_r]^{-1}[\Phi_{r\_dq0}]\right\} \quad (12)$$

$$[\Phi_{r\_dq0}] = [P_r][L_{rs}][P_r]^{-1}[i_{s\_dq0}] + [P_r][L_{rr}][P_r]^{-1}[i_{r\_dq0}] \quad (13)$$

After the simplification of the equations (10) and (12), the voltage equations of healthy-IM in dq-reference frame are expressed as follows:

$$\begin{bmatrix} v_{sd} \\ v_{sq} \end{bmatrix} = \begin{bmatrix} r_s & 0 \\ 0 & r_s \end{bmatrix}\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + P\begin{bmatrix} \Phi_{sd} \\ \Phi_{sq} \end{bmatrix} + \begin{bmatrix} 0 & -\omega_s \\ \omega_s & 0 \end{bmatrix}\begin{bmatrix} \Phi_{sd} \\ \Phi_{sq} \end{bmatrix} \quad (14)$$

$$\begin{bmatrix} 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_r & 0 \\ 0 & r_r \end{bmatrix}\begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} + P\begin{bmatrix} \Phi_{rd} \\ \Phi_{rq} \end{bmatrix} + \begin{bmatrix} 0 & -(\omega_s - p\Omega) \\ (\omega_s - p\Omega) & 0 \end{bmatrix}\begin{bmatrix} \Phi_{rd} \\ \Phi_{rq} \end{bmatrix} \quad (15)$$

Here: $P = \frac{\partial}{\partial t}$

Where:

$$\omega_s = \frac{\partial\theta_s}{\partial t},\quad \omega_r = \frac{\partial\theta_r}{\partial t},\quad \omega = \frac{\partial\theta}{\partial t} = p\Omega,\quad \omega_s = \omega_r + \omega$$

Ω is the mechanical speed, ω: electrical speed.

p: number of pole pairs.

The fluxes expressions in (11) and (13) become:

$$\begin{bmatrix} \Phi_{sd} \\ \Phi_{sq} \end{bmatrix} = \begin{bmatrix} L_s & 0 \\ 0 & L_s \end{bmatrix}\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} + \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} \quad (16)$$

$$\begin{bmatrix} \Phi_{rd} \\ \Phi_{rq} \end{bmatrix} = \begin{bmatrix} L_r & 0 \\ 0 & L_r \end{bmatrix}\begin{bmatrix} i_{rd} \\ i_{rq} \end{bmatrix} + \begin{bmatrix} M & 0 \\ 0 & M \end{bmatrix}\begin{bmatrix} i_{sd} \\ i_{sq} \end{bmatrix} \quad (17)$$

Where [4]:

$$L_s = l_{ls} + 1.5\,l_{ms},\quad L_r = l_{lr} + 1.5\,l_{ms},\quad M = 1.5\,l_{ms}$$

Motion and electromagnetic torque equations are given by the following formulas:

$$\begin{cases} \dfrac{\partial \Omega}{\partial t} = \dfrac{1}{J}\left(T_{em} - T_L - f v \Omega\right) \\ T_{em} = pM\left(i_{sq}\, i_{rd} - i_{sd}\, i_{rq}\right) \end{cases} \tag{18}$$

Where:

T_{em}, T_L, J and f_v are electromagnetic torque, load torque, inertia and viscous friction coefficient.

For obtaining IM-model in the stationary reference frame, in equations (14)-(15), ω_s is set to zero.

B. Model Of Three Phase IM under Open Phase Fault:

Assume that a failure occurs in phase "***c***" of the stator winding and has caused the cut out of that phase as shown in Fig.2 [5].

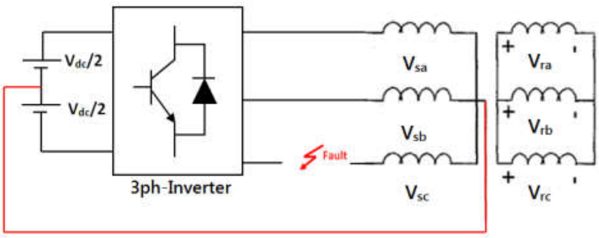


Fig.2. Three-phase IM drive system under open-phase fault condition.

In this unbalanced case, voltage and flux equations for faulty IM in the "**abc**" frame can be written as the following formulas:

- *For stator:*

$$\left[v_{s\_ab}\right]^{2\times 1} = \left[R_s\right]^{2\times 2}\left[i_{s\_ab}\right]^{2\times 1} + \frac{d}{dt}\left[\Psi_{s\_ab}\right]^{2\times 1} \tag{19}$$

$$\left[\Psi_{s\_ab}\right]^{2\times 1} = \left[L_{ss}\right]^{2\times 2}\left[i_{s\_ab}\right]^{2\times 1} + \left[L_{sr}\right]^{2\times 3}\left[i_{r\_abc}\right]^{3\times 1} \tag{20}$$

- *For rotor:*

$$\left[v_{r\_abc}\right]^{3\times 1} = \left[R_r\right]^{3\times 3}\left[i_{r\_abc}\right]^{3\times 1} + \frac{d}{dt}\left[\Psi_{r\_abc}\right]^{3\times 1} \tag{21}$$

$$\left[\Psi_{r\_abc}\right]^{3\times 1} = \left[L_{rr}\right]^{3\times 3}\left[i_{r\_abc}\right]^{3\times 1} + \left[L_{rs}\right]^{3\times 2}\left[i_{s\_ab}\right]^{2\times 1} \tag{22}$$

In equations (19)-(22) [3]:

$$\left[R_s\right]_{2\times 2} = r_s \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\left[L_{ss}\right]_{2\times 2} = l_{ls}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + l_{ms}\begin{bmatrix} 1 & -0.5 \\ -0.5 & 1 \end{bmatrix}$$

$$\left[L_{sr}\right]_{2\times 3} = l_{ms}\begin{bmatrix} cos(\theta) & cos\left(\theta + \frac{2\pi}{3}\right) & cos\left(\theta - \frac{2\pi}{3}\right) \\ cos\left(\theta - \frac{2\pi}{3}\right) & cos(\theta) & cos\left(\theta + \frac{2\pi}{3}\right) \end{bmatrix}$$

On the other hand, for the IM studied now, the well known (dq) transformation defined in (9) will not apply to the asymmetrical stator windings structure. So, the (dq) transformation for the stator must be redefined [1].

The *a-b-c* and its equivalent *d-q* axes for the stator and rotor fluxes of the faulty 3-phase induction motor in the stationary reference frame are represented in Fig. 3 [6]:

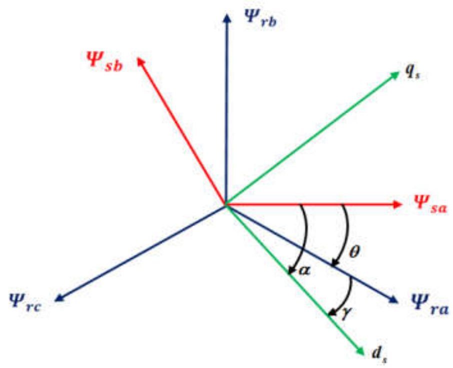


Fig.3. Rotor and stator winding's flux axes under open-phase fault.

In Fig.3, θ is the angle between ($\mathbf{a}_s$) and ($\mathbf{a}_r$) axes and α is the angle between ($\mathbf{a}_s$) and ($\boldsymbol{d}_s$) axes. Hence:

$$\gamma = \alpha - \theta \tag{23}$$

According to Fig.3, the normalized transformation for the stator and rotor components is obtained as [6]:

$$\left[P_s\right]^{Fault} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ 1 & 1 \end{bmatrix} \tag{24}$$

$$\left[P_r\right] = \sqrt{\frac{2}{3}}\begin{bmatrix} cos(\gamma) & cos\left(\gamma + \frac{2\pi}{3}\right) & cos\left(\gamma - \frac{2\pi}{3}\right) \\ sin(\gamma) & sin\left(\gamma + \frac{2\pi}{3}\right) & sin\left(\gamma - \frac{2\pi}{3}\right) \end{bmatrix} \tag{25}$$

Applying the transformation matrix $[\boldsymbol{P}_s]^{\text{Fault}}$ and $[\boldsymbol{P}_r]$ to the equations (19)-(22) , the new d-q model for the faulty motor in the stator reference frame is obtained as follows [1]:

- *Voltage equations:*

$$\begin{bmatrix} v_{sd} \\ v_{sq} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} r_s + L_{sd}P & 0 & M_d P & 0 \\ 0 & r_s + L_{sq}P & 0 & M_q P \\ M_d P & p\Omega M_q & r_r + L_r P & p\Omega L_r \\ -p\Omega M_d & M_q P & -p\Omega L_r & r_r + L_r P \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} \quad (26)$$

- *Flux equations:*

$$\begin{bmatrix} \Psi_{sd} \\ \Psi_{sq} \\ \Psi_{rd} \\ \Psi_{rq} \end{bmatrix} = \begin{bmatrix} L_{sd} & 0 & M_d & 0 \\ 0 & L_{sq} & 0 & M_q \\ M_d & 0 & L_r & 0 \\ 0 & M_q & 0 & L_r \end{bmatrix} \begin{bmatrix} i_{sd} \\ i_{sq} \\ i_{rd} \\ i_{rq} \end{bmatrix} \quad (27)$$

The new electromagnetic torque formula for faulty IM is given by the following equation:

$$T_{em} = p\left(M_q i_{sq}\, i_{rd} - M_d i_{sd}\, i_{rq}\right) \quad (28)$$

Where [1]:

$$L_{sd} = l_{ls} + 1.5\, l_{ms}, L_{sq} = l_{ls} + 0.5\, l_{ms}, M_d = 1.5\, l_{ms}, M_q = 0.5\sqrt{3}\, l_{ms}$$

3. Simulation and Results

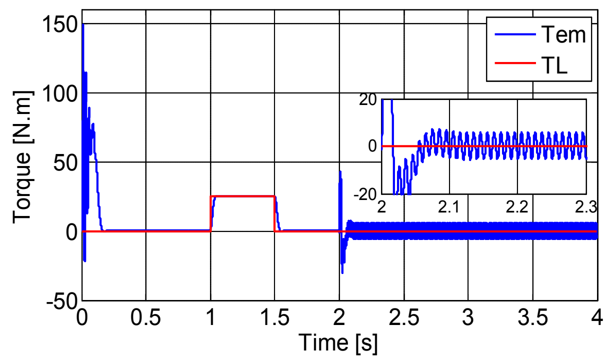
To study the dynamic behavior of three-phase induction machine under open stator phase fault, simulations by means of M-File/MATLAB are implemented using the model presented in this paper. The IM is fed by a 3-phase network voltage (the inverter will be used in the second part of this work). Runge-kutta (RK4) method is used to solve the IM dynamic equations for healthy and faulty operations.

Parameters of the simulation machine are given as follows:

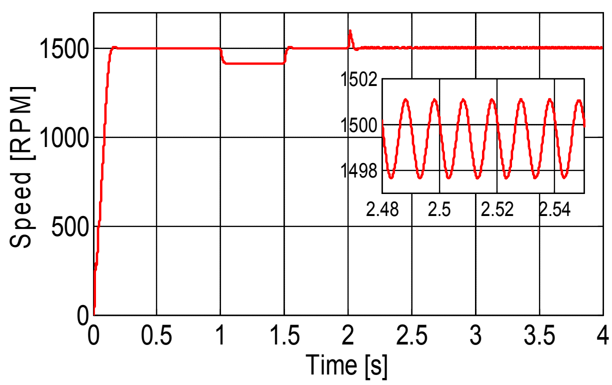
TABLE.1
ELECTRICAL AND MECHANICAL PARAMETERS OF THE ANALYSED 3-PHASE IM

Rotor type	squirrel-cage
Reference frame	stationary
Rated power	4 kW
Voltage	220/380 V
Frequency, *f*	50 Hz
Stator leakage inductance, l_{ls}	0.0068 H
Mutual inductance, *M*	0.15 H
Rotor leakage inductance, l_{lr}	0.0068 H
Stator resistance, *Rs*	1.2 Ω
Rotor resistance, *Rr*	1.8 Ω
Moment of inertia, *J*	0.05 kg.m^2
Load torque, *TL*	25 N.m
Number of pole pairs, *p*	2

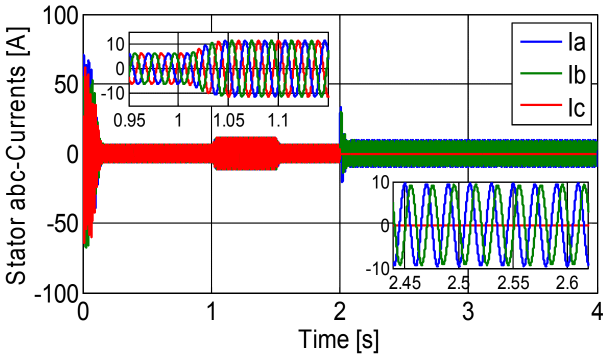
It is considered that from t = 0 s to t = 2s, the IM operates under healthy condition, then a fault is applied at t = 2s (one phase cut-off). As a result, the motor runs under open phase fault at t ≥ 2s. In addition, from t = 1s to t = 1.5s, a load torque equal to 25 N.m is applied.



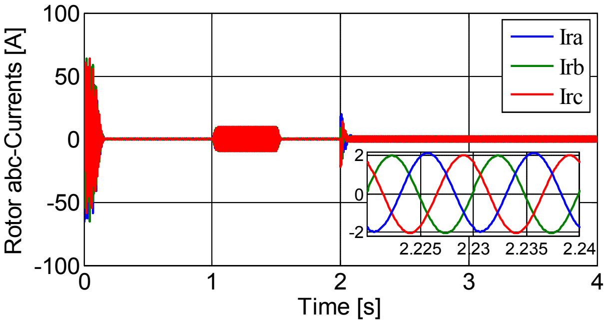
(a) Torque curve



(b) Speed curve



(c) Waveform of Stator currents



(d) Waveform of Rotor currents

Fig.4. Simulation results of IM under healthy and faulty operations; (a) Torque; (b) Speed; (c) abc-stator currents; (d) abc-rotor currents.

Waveforms of torque, speed, abc-stator currents and abc-rotor currents of simulated induction motor are shown in Fig.4. From these results it can be concluded that, the oscillations of torque and speed has increased when the appearance of fault.

On the other hand, we notice an increase in stator and rotor currents at fault in comparison with balanced operation.

4. Conclusions and Perspectives

In this paper, a modelling method of three-phase induction machine with open-circuit fault is presented. By this method, a faulty three-phase induction motor can be modelled as an equivalent unbalanced two-phase motor, which has the same structure of equations as the healthy three-phase induction motor.
In this method, the well-known d-q transformation will not apply to the asymmetrical stator windings structure. The d-q matrix transformation for the stator quantities should be redefined.
Simulation results are presented to show the dynamic behavior of the developed modeling method.
In future works, we will be interested by the vector control of a three phase IM under open phase fault.

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